

Look at the behaviour of  $f(x) = y$  when we fix a  $y$  and investigate the acceptable range of values  $x$  can take while still being within the line of the graph. First fix  $\Delta$  to be the cutoff value where we set a pixel to on given the following holds

$$|f(x) - y| < \Delta$$

When we fix  $y = b$  for some constant  $b$ , then for  $x$  satisfying

$$b - \Delta < f(x) < b + \Delta$$

The pixel is on. So this gives a range of  $2\Delta$  for values of  $f(x)$ . So that the range of values for  $x$  will often vary along the length of the function.

For a particular example look at the function  $f(x) = x^3$  which has an inverse function  $x^{\frac{1}{3}}$ .

$$\begin{aligned} b - \Delta < x^3 < b + \Delta \\ (b - \Delta)^{\frac{1}{3}} < x < (b + \Delta)^{\frac{1}{3}} \end{aligned}$$

Giving values of  $x$  a range of

$$|(b - \Delta)^{\frac{1}{3}} - (b + \Delta)^{\frac{1}{3}}|$$

Which gives a large cutoff for  $b$  close to zero and a smaller cutoff as  $b$  increases.